SECOND ORDER BLIND SOURCE SEPARATION BY RECURSIVE SPLITTING OF SIGNAL SUBSPACES

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ABSTRACT

We present an approach to blind source separation based on delayed correlations. This method recursively splits separation space into subspaces spanned by groups of sources. The inner loop consists of repeated application of a standard eigenvalue decomposition. When the number of sources is large this algorithm is significantly faster than joint diagonalization of cross-covariance matrices.

1. INTRODUCTION

We present an approach to blind source separation based on second order statistics. When each source has a broad auto-correlation function (as is the case with sounds or EEG/MEG signals), second order methods can provide higher quality separation than methods which assume the sources to be white, for instance algorithms based on instantaneous higher-order moments.

In contrast to other second order techniques [2, 4, and references therein], our method is based on repeated eigenvalue decomposition, and is therefore significantly faster than previous algorithms for large problems.

2. DERIVATION

Let $x(t)$ be an N-dimensional vector of sensor signals which is an instantaneous linear mixture of N unknown independent sources $s_j(t)$

$$x(t) = As(t) \tag{1}$$

The problem is to estimate the unknown mixing matrix A, up to a permutation and scaling of its rows. Second order methods are based on the observation that statistically independent sources should be uncorrelated, even if one is delayed relative to another. This means that the cross-covariance matrices

$$R_{s(\tau)} = E\{s(t)s(t-\tau)^T\} = \Lambda_\tau \tag{2}$$

will be diagonal. The solution of the blind source separation problem is indeterminate with respect to scaling of the sources. We therefore assume without loss of generality that the sources have unit variance. Taking into account their independence, we get the identity of the covariance matrix

$$R_{s(0)} = I. \tag{3}$$

We will also suppose that the raw data is *presphered*, in other words that a linear transformation has already been applied to the data to ensure that $R_{x(0)} = I$. In this presphered coordinate system, equations (1) and (3) force A to be be orthogonal:

$$R_{x(0)} = AR_{s(0)}A^T = AA^T = I$$

The cross-covariance matrix of x can be expressed using (2) as

$$R_{x(\tau)} = AR_{s(\tau)}A^T = A\Lambda_\tau A^T.$$

We can see that the mixing matrix A is a part of the eigenvalue decomposition of $R_{x(\tau)}$, which gives a way to estimate A. However, if the correlation matrix $R_{x(\tau)}$ has some inaccuracy, and a few of the eigenvalues are close to each other, there can be a large error in the estimates of the corresponding eigenvectors. This problem is addressed in [2, 4, 5] by the use of joint diagonalization of a few matrices $R_{x(\tau)}$ with different delays τ. This method generally provides high quality separation, but the computational burden increases rapidly as the number of sources and cross-covariance matrices grows.

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We consider a different approach which gives a comparable quality of source separation, but is much faster on large problems. This method is based on the following observation. Let $\lambda_1, \lambda_2, \ldots, \lambda_k, \lambda_{k+1}, \ldots, \lambda_N$ be eigenvalues sorted in descending order. Suppose that the gap between two of them, $\lambda_k - \lambda_{k+1}$, is large. Then the corresponding subspaces of the eigenvectors $\mathcal{U}_1 = \text{Span}\{u_1, u_2, \ldots, u_k\}$ and $\mathcal{U}_2 = \text{Span}\{u_{k+1}, \ldots, u_N\}$ are estimated accurately, despite the fact that the eigenvectors inside each group may have significant errors.

Let the columns of the matrices U_1 and U_2 be the eigenvectors of the first and the second groups above, and let the matrix $U = [U_1 U_2]$ include them both. The observed signal can be expressed as

$$x = [U_1 U_2] \begin{bmatrix} z^1 \\ z^2 \end{bmatrix}, \qquad (4)$$

where each vector signal $z^i(t)$ corresponds to a group of sources $s^i(t)$, possibly mixed by

$$z^i(t) = \bar{A}_i s^i(t), \quad i = 1, 2 \qquad (5)$$

We can solve equation (4) with respect to $z^i(t)$, multiplying both sides by U^T and taking into account its orthogonality

$$\begin{bmatrix} z^1 \\ z^2 \end{bmatrix} = \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} x$$

or

$$z^1 = U_1^T x \qquad\qquad z^2 = U_2^T x$$

In this way we end up with two smaller separation problems (5). Each of them can be solved recursively, by the same routine that was used for the original problem (1), using the new covariance matrices

$$R_{z^i(\tau)} = U_i^T R_{x(\tau)} U_i, \quad i = 1, 2$$

This yields the following recursive method of subspace separation.

Recursive description of the algorithm

1. Initialize the matrix $B = I$
2. Among the covariance matrices[1] $R_{x(\tau_1)}$, $R_{x(\tau_2)}$, ..., $R_{x(\tau_M)}$ find the one with the maximal gap between neighboring sorted eigenvalues. This will split the eigenvectors of this matrix into two sets, U_1 an U_2.

[1] When the covariance matrices are estimated from the data, we have to symmetrize them $R_{x(\tau)} = \frac{1}{2}(R_{x(\tau)} + R^T_{x(\tau)})$, in order to provide exact orthogonality of their eigenvectors.

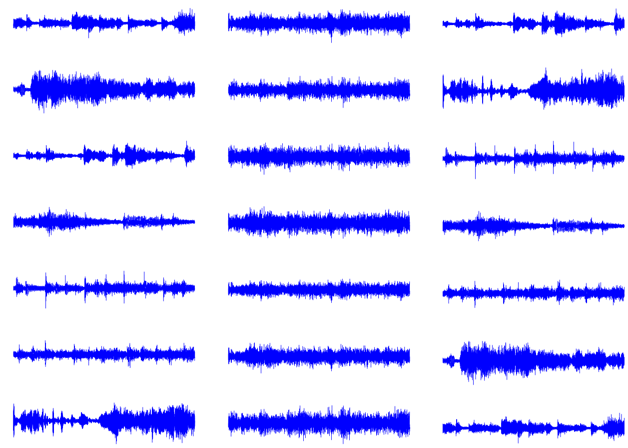

Figure 1: Separation of musical recordings taken from commercial digital audio CDs (five second fragments).

3. Compute the matrices $B_i = U_i^T B$, $i = 1, 2$, which map from signal space into source subspaces
4. Compute two groups of covariance matrices which correspond to these sources subspaces: $R_{z^i(\tau)} = U_i^T R_{x(\tau)} U_i,\ i = 1, 2.$
5. Recurse through steps 2–5 for each subspace, using $R_{x(\tau)} \equiv R_{z^i(\tau)}$ and $B \equiv B_i$. When the dimension of a subspace is one, then instead of recursing, put B_i into a row of separation matrix W.
6. After the recursion has terminated, all subspaces will be of dimension one. At the point, the separation matrix W will be fully calculated, and one can calculate the separated sources $s(t) = Wx(t)$

3. COMPUTATIONAL EXPERIMENTS WITH MUSICAL SOUND SOURCES

We digitally mixed seven 5-second fragments taken from commercial digital audio music CDs. Each of them included 40k samples after averaging the two channels and down-sampling by a factor of five. See Figure 1.

The set of delays τ used for the cross-covariance matrices was chosen to cover reasonably wide interval. It was, measured in samples:

$$\tau = 1, 2, \ldots, 9, 10, 12, 14, \ldots, 18, 20, 25, 30, \ldots, 95, 100$$

We separated the data by the method proposed above, and for comparison by the SOBI algorithm [2],

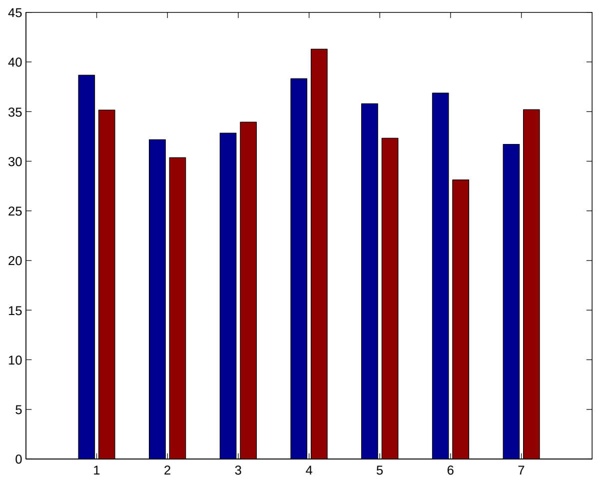

Figure 2: Signal to noise ratio (dB) of 7 separated sources. With each column SOBI is on the left and the algorithm presented here on the right.

which uses joint diagonalization [5] of the same cross-covariance matrices.

The signal-to-noise ratio in dB for all the separated sources is shown in Figure 2 (in each double-column the results of SOBI are shown on the left, of our new method on the right.) As can be seen, the accuracy of two algorithms is comparable. Each of them took less then 0.5 sec on a 300MHz AMD K6-III processor. When we did a similar comparison for 122-channel MEG data, with the same set of delays, the new algorithm took 5 min, while SOBI took 10 hours. This shows that the new algorithm scales well, as compared to SOBI.

4. FUTURE RESEARCH

We leave one more possibility for future research. It often happens at some stage that all cross-covariance matrices of the subspace under consideration have small maximal eigenvalue gaps. This means that all the sources in corresponding group have similar power spectra. In this case second-order methods may be inadequate. However, one could use other blind source separation methods within this subspace [1, 3, 6, 7].

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