BLIND SEPARATION OF NON-CIRCULAR SOURCES

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ABSTRACT

Blind Source Separation is now a well known problem. When a priori informations about the propagation or the geometry of the array are not available, the model can be generalized to a blind source separation model. Il supposes the statistical independence of the sources and their non-gaussianity. In this paper, we focus on an algorithm, called Canonical Correlation Analysis, based on the use of second order statistics.

1. CANONICAL CORRELATION ANALYSIS

The Canonical Correlation Analysis is a method of treatement which allows to study the correlation between two sets of data.

We can have one set of data k fonction of the observed signals.

$$k = g\left[x\right] \tag{1}$$

In blind source separation, and in particular when we are interested in anti-jamming processing, we divide the received signals into source0 signals and noise signals. The second set of data k is get from the observed signals x of the antenna. This processing is selected to keep the signal of interest :

$$\begin{aligned} x &= x_{interest} + x_{noise} \\ k &= k_{interest} + k_{noise} \\ R_{xk} &= E\left[xk^H\right] = E\left[x_{interest} {k_{interest}}^H\right] \end{aligned} \tag{2}$$

The Canonical Correlation Analysis can be divided in several steps.

The first step is to write the two whitened sets of data :

$$\begin{aligned} \Xi_x &= R_x^{-1/2} x \\ \Xi_k &= R_k^{-1/2} k \end{aligned} \tag{3}$$

with : $E\left[\Xi_x X i_k^H\right] = R_x^{-1/2} R_{xk} R_k^{-1/2}$
We can find the eigenvalues of this matrix :

$$E\left[\Xi_x \Xi_k^H\right] = U\Sigma^2 V^H \tag{4}$$

If we develop two new matrix :

$$\alpha = U^H \Xi_x \tag{5}$$

and

$$\beta = V^H \Xi_k \tag{6}$$

then we can say that α has all of the information on Ξ_k which can be obtained from Ξ_x and mutually, β has all of the information on Ξ_x which can be obtained from Ξ_k.

So we resolve $E\left[\alpha\beta^H\right] = \Sigma^2$ under : $E\left[\alpha\alpha^H\right] = I = E\left[\beta\beta^H\right]$.

Suppose that we have two sets of data x and k available.

The Canonical Correlation Analysis consists in defining two matrix W_x and W_k in order to $W_x^H x$ and $W_k^H k$ must be the more correlated.

So we have :

$$\begin{aligned} \alpha &= W_x^H x \\ \beta &= W_k^H k \end{aligned} \tag{7}$$

The Canonical Correlation Analysis minimizes the criterion :

$$\Phi(W_k, W_x) = E\left[|{W_k}^H k - {W_x}^H x|^2\right] \quad (8)$$

under :

$${W_k}^H R_k W_k = 1 \quad (9)$$

and

$${W_x}^H R_x W_x = 1 \quad (10)$$

The minimization can be written :

$$\begin{aligned} \Phi(W_k, W_x) &= trace \\ & \left[\begin{array}{l} W_k^H R_k W_k + W_x^H R_x W_x - \\ W_k^H R_k x W_x - W_x^H R_x k W_k \end{array} \right] \end{aligned} \quad (11)$$

To minimize $\Phi(W_k, W_x)$, we must derive from each component of W_k, W_x and use Lagrange operations Λ et Δ which are not discussed in our paper..

We have now two equations :

$$R_{xk} W_k = R_x W_x \Lambda \quad (12)$$

$$R_{kx} W_x = R_k W_k \Delta \quad (13)$$

We modify the equation, (multiplication by $R_{kx}^{-1} R_{kx}$) we can see :

$$\begin{aligned} R_{xk} W_k &= R_x R_{kx}^{-1} R_{kx} W_x \Lambda \quad (14) \\ &= R_x R_{kx}^{-1} R_k W_k \Delta \Lambda \end{aligned}$$

If we are interested in W_x , we can have the dual equation.

If we call $T = \Delta\Lambda$, then :

$$R_k^{-1} R_{kx} R_x^{-1} R_{xk} W_k = W_k T \quad (15)$$

If we multiply by $R_k^{1/2}$, we have the equation :

$$R_k^{-1/2} R_{kx} R_x^{-1/2} R_x^{-1/2} R_{xk} R_k^{-1/2} R_k^{1/2} W_k = R_k^{1/2} W_k T \quad (16)$$

With $R_k^{-1/2} R_{kx} R_x^{-1/2} = D$, then :

$$D D^H \tilde{W}_k = \tilde{W}_k T \quad (17)$$

with $\tilde{W}_k = R_k^{1/2} W_k$

So we can find the eigenvalues and eigenvectors of D. We choose the L eigenvectors U_1 corresponding to L higher eigenvalues.

The matrix W_k is now :

$$W_k = {R_k}^{1/2} U_1 \quad (18)$$

The argument is the same if we are interested in the matrix W_x.

2. APPLICATIONS

We take the model without noise : $x = As$. The matrix A have the SVD : $A = U\Sigma V$

2.1. SOBI

If the second sets of data can be deduced from the first ones with the addition of a delay on the signal :

$$k = As(t - \tau) \quad (19)$$

and

$$x = As(t) \quad (20)$$

We can write : $R_x = R_k = U\Sigma^2 U^H$.
We can have :

$$\begin{aligned} \Xi_x &= R_x^{-1/2} x = V s(t) \quad (21) \\ \Xi_k &= R_k^{-1/2} k = V s(t - \tau) \end{aligned}$$

with : $R_x^{-1/2} = R_k^{-1/2} = \Sigma^{-1} U^H$
We have also :

$$E\left[\Xi_x {\Xi_k}^H\right] = V R_s(\tau) V^H \quad (22)$$

To specify V, *Belouchrani* in SOBI [1] choose to make a joint diagonalization of a set of matrix using second order statistics. This approach can be compared with the *Cardoso* and *Souloumiac* method for the *Jade* algorithm [2].

The estimated V allows to form the estimated mixing matrix $\hat{A}$:

$$\hat{A} = R_x^{1/2} V \quad (23)$$

and the estimated outputs are :

$$\widehat{s}(t) = V^H R_x^{-1/2} x \quad (24)$$

2.2. Non-circular Source Separation

In our case, the signals *(BPSK)* are non-circular and the noise signals are circular. If the interference signals are $j(t)$ and the BPSK are $s(t)$, we can see that :

$$E\left[s(t)^2\right] \neq 0 \quad (25)$$

and

$$E\left[j(t)^2\right] = 0 \quad (26)$$

The signals *(BPSK)* are non-circular, if we want to eliminate the circular interferences, we can use for $k(t)$ the conjugate of $x(t)$:

$$k(t) = x(t)^* \quad (27)$$

The model is always $x = As$:

$$k(t) = A^* s(t)^* \quad (28)$$

We look at the matrix A which can be divided in eigenvalues and we can write the conjugate A noted A^* :

$$A^* = U^* \Sigma V^* \tag{29}$$

The correlation matrix of the source signals is :

$$R_x = E\left[x(t)x(t)^H\right] = U\Sigma^2 U^H \tag{30}$$

Now the correlation matrix of the set of data $k(t)$ can be written :

$$R_k = E\left[k(t)k(t)^H\right] = E\left[x(t)^* k(t)^T\right] = U^* \Sigma^2 U^T \tag{31}$$

If we have the whitened sets of data :

$$\begin{aligned} \Xi_x &= R_x^{-1/2} x = \Sigma^{-1} U^H x = V s(t) \\ \Xi_k &= R_k^{-1/2} k = \Sigma^{-1} U^T x = V^* s^*(t) \end{aligned} \tag{32}$$

with :

$$R_x^{-1/2} = \Sigma^{-1} U^H \tag{33}$$

and

$$R_k^{-1/2} = \Sigma^{-1} U^T \tag{34}$$

We have :

$$E\left[\Xi_x {\Xi_k}^H\right] = V E[ss^T] V^T \tag{35}$$

The matrix $E[ss^T]$ only contains the informations on non-circular signals. The SVD factoring of $E[ss^T]$ allows to estimate V and to find only the non-circular signals of the mixing.

Th estimation of V allows to have the estimated mixing matrix $\hat{A}$:

$$\hat{A} = R_x^{1/2} V \tag{36}$$

This research of the mixing matrix can be qualified a *blind separation* because no information on antenna, on propagation or on signals is necessary to have the 'filter'. The noise signals must be circular to be rejected by this algorithm [3].

One of the applications of this algorithm is the subject of a patent registered with Thomson-CSF.

3. RESULTS

3.1. Adaptive Antenna

The antenna is an *MSLC (Multiple Sidelobe Canceller) antenna*, which means that we can have one main antenna and some auxiliary elements. Indeed, for the supervision of some particular space aeras, we use this kind of antenna which allows to focus on the main antenna the information on the source signal while supervising areas likely to have some jamming signals.

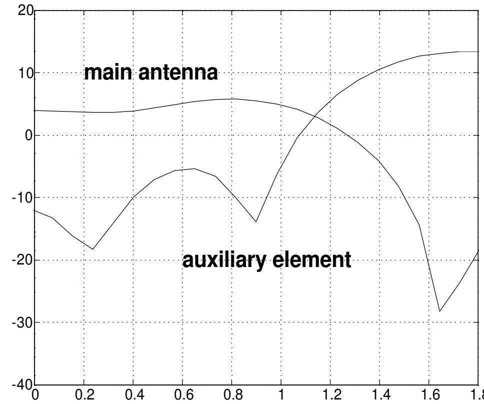

Figure 1: MSLC Antenna

If we consider the sectional elevation, we can recognize on figure 1 the main antenna and an auxiliary element. The main antenna has a constant value (3-4 dB) between 0°and 1°(angle of sight) and the auxiliary element has some variations between these angles of sight.

The source signal and the jamming sources are :

- **1 source signal** located at 0° (angle of sight), 0° (yaw angle) and with power $20dB$.
- **2 gaussian jammers** located one at 0° (angle of sight) and 1.5° (yaw angle) with power $20dB$ and other at 1.7° (angle of sight) and 0° (yaw angle) with power $20dB$.
- **gaussian noise** with power $0dB$.

This kind of situation is good for the classical treatement (anti-jamming with MSLC). The source signal is located in the sidelobe of the main antenna and the two jammers are located in the middle of the auxiliary elements. Now we can compare the performances of the different algorithms.

3.2. Performances

It's necessary to study the performances of the two algorithms (MSLC and Canonical Correlation Analysis) when one of the jammers will move and its power will change.

When one of the jammers moves, the performances of the differents algorithms can be evaluated :

– – keeping the two others fixed.

– changing the last jammer initially located at 0^o (yaw angle) along the decreasing angles of sight (variation from 1.7^o to 0^o).

The power of the moving jammer is fixed to $20dB$ and increases to $50dB$.

– **MSLC treatement**

On figure 2, lots of elements allow us to verify :

– – the Signal to Noise and Interferences Ratio (SINR) becomes weak when the jammer comes near the source signal.

– the SINR becomes weak when the power of the jammer decreases.

These two observations are easily explainable.

For the first one, this is inevitable whichever algorithms we use. This waste of *SINR* is predictable : if we look at the figure 1, we can see the jammer entering in the main antenna from 1° (angle of sight), and the *SINR* variation follows the auxiliary element.

The second observation is the result of **the self-jamming of the source signal**. In fact, when the power of the jammer signal is weak, the MLSC algorithm takes the source signal as a jammer and it tries to eliminate it. That is why we call this, self-jamming.

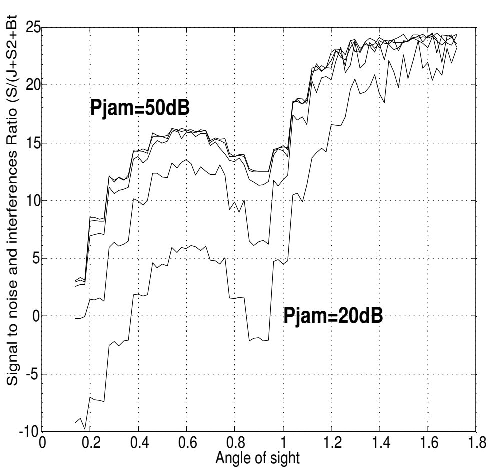

Figure 2: SINR with MSLC Algorithm

– **Canonical Correlation Analysis**

If we make the same experience with the Canonical Correlation Analysis, we can see on figure 3 that the Signal to Noise and Interferences Ration does not depend on the power of the jammer. The self-jamming of the source signal has disapeared. Whichever the jammer power, the SINR only depends on the auxiliary element.

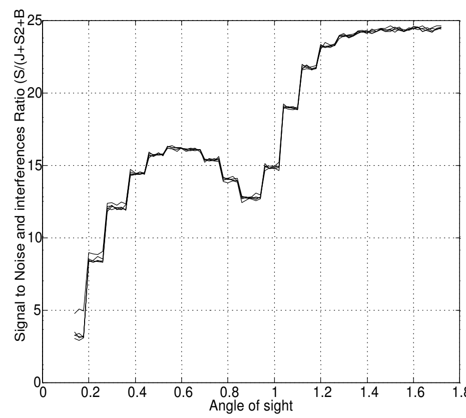

Figure 3: SINR with Canonical Correlation Analysis

4. CONCLUSION

If we can have specific information on the source signals, it is better to use methods only based on the use of second-order statistics. These methods are less expansive for the calculation than higher order statistics.

The BPSK signals are non-circular while the jammer sources are gaussian and circular. For Separation Sources, the best technique is the Canonical Correlation Analysis, the results show that this method is effective to avoid the self-jamming of the source signal.

In the case of a MSLC antenna, the performances with Correlation Canonical Analysis are the same as these with JADE algorithm [2] using higher-order statistics [3].

5. REFERENCES

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